Errata for "EKOR strata for Shimura varieties with parahoric level structure"

Xu Shen, Chia-Fu Yu, and Chao Zhang

Here are some corrections of our paper *EKOR strata for Shimura varieties with parahoric level structure*, Duke Math. J. 170, no. 14 (2021). We would like to thank Manuel Hoff and David Loeffler for pointed out these two problems respectively.

We will use notations as in our paper.

1. As pointed by Manuel Hoff, the "commutative diagram" in the middle of page 3191 is NOT commutative. Otherwise, the Iwahori KR strata in M^{loc} attached to $x \in U$ (introduced in page 3190) will always be a point. However, our proof still works with slight modifications, i.e. if one argues étale locally as we are going to explain.

   We start with the sentence before the non-commutative diagram. By the proof of [1, Corollary 4.2.13], shrinking U (an étale neighborhood of $x \in \mathrm{Sh}_K$) and changing the section a if necessary, we assume that the composition $a : U \to \mathrm{Sh}_K^{(1,0)\square} \to M^{\mathrm{loc}}$ is induced by an étale morphism of schemes. In the following, we slightly abuse of notation to view $x \in U$ and $x^{\square} \in U \times L^m\mathcal{G}$ (different from those in page 3190). Removing the remaining words in the paragraph containing the diagram, we are going the prove that the composition

   $$a^{\square}(m,1) : U \times L^m\mathcal{G} \to \mathrm{Sh}_K^{(1,0)\square} \to M^{\mathrm{loc},(1)-\mathrm{rdt}}$$

   is perfectly smooth at an open neighborhood of some $x^{\square} = (x, g) \in U \times L^m\mathcal{G}$ lifting x. The morphism $\mathrm{Sh}_K^{(1,0)\square} \to M^{\mathrm{loc}}$ is equivariant for the natural projection $L^m\mathcal{G} \to \mathcal{G}_0$, replacing $U \subseteq U \times L^m\mathcal{G}$ by $U \times \{g\}$, we assume without lose of generality that $x^{\square} = (x, \mathrm{id})$. Now it suffices to show that

   $$\widetilde{a^{\square}(m,1)} : (U \times L^m\mathcal{G}) \times_{M^{\mathrm{loc}}} U \longrightarrow M^{\mathrm{loc},(1)-\mathrm{rdt}} \times_{M^{\mathrm{loc}}} U$$

   is perfectly smooth near $x^{\square}$, as $U \to M^{\mathrm{loc}}$ is étale.

   In the last paragraph, we keep the first two sentences and start to change from the third. The composition $a : U \to M^{\mathrm{loc}}$ descends to an étale morphism $a' : U' \to M^{\mathrm{loc,PZ}}$ by assumption (here $M^{\mathrm{loc,PZ}}$ is the special fiber of the Pappas-Zhu local model). Let $U'^{(1)}$ be the trivial $\mathcal{G}_0^{\mathrm{rdt}}$-torsor over U'. The above morphism $\widetilde{a^{\square}(m,1)}$ is the perfection of

   $$f : (U' \times L_p^m\mathcal{G}) \times_{M^{\mathrm{loc,PZ}}} U' \longrightarrow U'^{(1)}, \quad ((u,g), gu) \mapsto ga(m,1)(u)\pi_{m,1-\mathrm{rdt}}(\sigma(g)^{-1}).$$

Here the action of g is via its image in $\mathcal{G}_0$. Noting that f is a morphism of smooth families over U', we only need to check that f_x, the morphism on fibers over x via the second projections to U', induces a surjection of tangent spaces at $x^\square$. The fiber of x in $(U' \times L_p^m\mathcal{G}) \times_{M^{\mathrm{loc,PZ}}} U'$ is the smooth subvariety defined by

$$gz = x, \quad z \in U', g \in L_p^m\mathcal{G}.$$

The tangent spaces at $x^\square = (x, \mathrm{id})$ is

$$\{(g^{-1}x, g) \mid g \in \mathrm{Lie}(L_p^m\mathcal{G})\},$$

which could be identified with $\mathrm{Lie}(L_p^m\mathcal{G})$, so the tangent map of f_x at $x^\square$ coincides with the one at id of

$$L_p^m\mathcal{G} \to \mathcal{G}_0^{\mathrm{rdt}}, \quad g \mapsto \pi_{m,1-\mathrm{rdt}}(g \cdot \sigma(g)^{-1}).$$

The last two sentences on page 3191 then work without change.

2. As pointed out by David Loeffler, there are some typos on page 3218, in the list of KR and EKOR in part (4). For KR types, the first one should be $[\tau]_J = \{\tau, s_1\tau\}$; for EKOR, the first one should be τ; and for p-rank, the forth one should be $0, 2$. Here is the modified list.

(4) Case $J = \{0, 2\}$ (Siegel parahoric level) and $W_{J^c} = \langle s_1 \rangle$. Using $\tau s_1 = s_1\tau$, we get

KR types	EKOR	dim	p-rank
$[\tau]_J = \{\tau, s_1\tau\}$,	τ	0	0
$[s_{21}\tau]_J = [s_2\tau]_J = \{s_2\tau, s_{21}\tau s_{12}\tau\}$,	$s_2\tau$, $s_{21}\tau$	1, 2	0. 1
$[s_{01}\tau]_J = [s_0\tau]_J = \{s_0\tau, s_{10}\tau, s_{01}\tau\}$,	$s_0\tau$, $s_{01}\tau$,	1, 2	0, 1
$[s_{021}\tau]_J = [s_{02}\tau]_J = \{s_{02}\tau, s_{021}\tau, s_{102}\tau\}$,	$s_{02}\tau$, $s_{021}\tau$,	2, 3	0, 2
$[s_{212}\tau]_J = \{s_{212}\tau\}$,	$s_{212}\tau$,	3	2
$[s_{010}\tau]_J = \{s_{010}\tau\}$,	$s_{010}\tau$	3	2.

References

[1] M. KISIN and G. PAPPAS, *Integral models of Shimura varieties with parahoric level structure*, Publ. Math. Inst. Hautes Études Sci. 128 (2018), 121-218.