Derivation of the Vlasov equation from quantum many-body
Fermionic systems with singular interaction

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Abstract
We consider the combined mean-field and semiclassical limit for a system of the $N$ fermions interacting through singular potentials. We prove the uniformly in the Planck constant $h$ propagation of quantum moments for the Hartree–Fock equation with singular pair interaction potential of the form $|x−y|^{−a}$, including the Coulomb interaction. Using these estimates, we obtain quantitative bounds on the distance between solutions of the many-body Schrödinger equation and solutions of the Hartree–Fock and the Vlasov equations in Schatten norms. For $a \in (0, \frac{1}{2})$, we obtain global-in-time results when $N^{−\frac{1}{2}} \ll h < CN^{−\frac{1}{3}}$. In particular, it leads to the derivation of the Vlasov equation with singular potentials. For $a \in (\frac{1}{2}, 1]$, our results hold only on a small time scale $t \sim h^{a−\frac{1}{2}}$, or with an $N$-dependent cutoff. The talk is based on our recent works in [1, 2, 3]. This is a joint work with Laurent Lafleche and Chiara Saffirio. The talk will be delivered in English and is meant for the general audience.

References