Title: Multiplicity one for the mod p cohomology of Shimura curves

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Abstract: At present, the mod p (and p-adic) local Langlands correspondence is only well understood for the group $\mathrm{GL}_2(\mathbb{Q}_p)$. One of the main difficulties is that little is known about supersingular representations besides this case, and we do know that there is no simple one-to-one correspondence between representations of $\mathrm{GL}_2(K)$ with two-dimensional representations of $\mathrm{Gal}(\overline{K}/K)$, at least when $K/\mathbb{Q}_p$ is (non-trivial) finite unramified.

However, the Buzzard-Diamond-Jarvis conjecture and the mod p local-global compatibility for $\mathrm{GL}_2/\mathbb{Q}$ suggest that this hypothetical correspondence may be realized in the cohomology of Shimura curves with characteristic p coefficients (cut out by some modular residual global representation $\bar{r}$). Moreover, the work of Gee, Breuil and Emerton-Gee-Savitt show that, to get information about the $\mathrm{GL}_2(K)$-action on the cohomology, one could instead study the geometry of certain Galois deformation rings of the p-component of $\bar{r}$. In a work in progress with Haoran Wang, we push forward their analysis of the structure of potentially Barsotti-Tate deformation rings and, as an application, we prove a multiplicity one result of the cohomology at full congruence level when $\bar{r}$ is reducible generic *non-split* at p. (The semi-simple case was previously proved by Le-Morra-Schraen and by ourselves.)