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# GENERALIZED SERRE CONJECTURES, LOCAL-GLOBAL COMPATIBILITY AND THE $p$ -ADIC LOCAL LANGLANDS PROGRAM

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We discuss about the generalization of the weight part of Serre conjecture for  $\mathbf{GL}_3$  and how these conjectures are related to the mod  $p$  and  $p$ -adic local Langlands program.

Fix  $K/\mathbf{Q}_p$  and let  $\bar{\rho} : G_K \rightarrow \mathbf{GL}_n(\bar{\mathbf{F}}_p)$  be a continuous Galois representation. When  $\bar{\rho}$  is ordinary maximally non-split and Fontaine-Laffaille there is a family of parameters obtained from the Frobenius action on the Fontaine-Laffaille module which determine uniquely the isomorphism class of  $\bar{\rho}$ . Assume that  $n = 3$ ,  $K = \mathbf{Q}_p$  and  $\bar{\rho}$  is a local parameter of a modular global Galois representation  $\bar{\tau} : G_F \rightarrow \mathbf{GL}_n(\bar{\mathbf{F}}_p)$ , where  $F$  is a suitable CM field. Write  $H^0[\mathfrak{m}_{\bar{\rho}}]$  for the corresponding Hecke isotypical space of the mod  $p$  cohomology of  $U(3)$ -arithmetic manifolds with infinite level at  $p$ .

Assuming the weight part of Serre conjecture, the action of certain distribution algebra operators on  $H^0[\mathfrak{m}_{\bar{\rho}}]$  lets us recover the family of Fontaine-Laffaille parameters. In particular the smooth  $\mathbf{GL}_3(\mathbf{Q}_p)$ -representation  $H^0[\mathfrak{m}_{\bar{\rho}}]$  determines completely the local Galois parameter  $\bar{\rho}$ . (This is joint work with F. Herzig and D. Le).

In the second talk we focus on Breuil's lattice conjecture, which can be seen as a deepening of Serre's modularity conjecture. The conjecture concerns the integral structures which are cut out by the *completed* cohomology (an integral characteristic zero version of the space  $H^0[\mathfrak{m}_{\bar{\rho}}]$  above) on the tame types appearing in the space of cuspidal automorphic forms on  $U(3)$ -arithmetic manifolds.

We will explain how Serre's conjecture, together with some involved representation theory of finite groups of Lie type and algebraic Weyl modules, implies Breuil's, showing in particular that the integral structure cut out by cohomology on tame automorphic types depends only on the  $p$ -adic Galois parameter associated to Hecke eigensystems lifting  $\mathfrak{m}_{\bar{\tau}}$ . (This is joint work with D. Le, B. Levin and V-B. Le Hung).