Workshop on

Representation Theory: Cohomology and Support

Morningside Center of Mathematics, Chinese Academy of Sciences
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Abstract

An introduction to tensor triangular geometry

Paul Balmer (University of California, Los Angeles)

The lectures will start with a general introduction to tensor triangulated categories, giving axioms and examples. Then, I will present the basics of TT-geometry: the spectrum, the classification theorem, etc, and provide examples. Finally, I shall give examples of applications, including the generalized Carlson theorem, the gluing of endotrivial representations, etc.

Support varieties for finite groups and group schemes

Jon F. Carlson (University of Georgia)

In these lectures, I plan to present an introduction to the theory of support varieties with a sketch of at least one application. This subject, which began in the representation theory of finite groups, has been around for more than thirty years, and has become expanded to include finite group schemes and other rings. It has been integrated into parts of homotopy theory and commutative algebra. The three lectures will concentrate roughly on the following topics. In each lecture I will try to present many examples.

1. The structure of cohomology rings of finite groups – how we compute them, their ring spectra, and Quillen's Dimension Theorem.
2. Support varieties and their properties – the rank variety, tensor product theorem, connectedness theorem.
3. An application to the classification of endotrivial modules.

Support for complexes, Hopkins' theorem for perfect complexes, and Benson–Carlson–Rickard theorem

Srikanth B. Iyengar (University of Nebraska–Lincoln)

The three lectures focuses on the following topics.

(1) Support for complexes over commutative rings.
(2) Hopkins' theorem for perfect complexes over commutative rings, and some applications to commutative algebra.
(3) Benson–Carlson–Rickard theorem on thick subcategories, only for elementary abelian 2-groups.

An introduction to triangulated categories in representation theory

Henning Krause (Bielefeld University)

Triangulated categories arise in representation theory as stable or derived categories. These lectures provide a gentle introduction to this subject. I will start by reviewing some of the classical results (the results of Beilinson and Bernstein–Gelfand–Gelfand from 1978), because the main techniques and constructions are already there. Then I move to locally finite triangulated categories, because this is a class of triangulated categories where one has a well developed Auslander–Reiten theory. Finally, I'll talk about a classification of thick subcategories for derived categories of hereditary algebras, based on exceptional sequences and braid group actions.

Hochschild Cohomology and Support Varieties

Sarah Witherspoon (Texas A&M University)

In these lectures, we will start with a general introduction to Hochschild cohomology, including its meaning in low degrees and examples. We will discuss properties of Hochschild cohomology of finite dimensional algebras, particularly group algebras, Hopf algebras, and quantum groups. We will define support varieties using Hochschild cohomology, under some finiteness conditions, and discuss current conjectures and applications to representation type.

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Frobenius categories, revisited

Xiaowu Chen (University of Science and Technology of China)

A surprising result of Kussin–Lenzing–Meltzer states that the (graded) submodule category is a factor category of the category of vector bundles on the weighted projective line of type (2,3,p). However, their proof is rather puzzling. We will reduce their proof partially into two general nonsenses of Frobenius categories.

On stratifications of derived module categories

Seffen König (University of Stuttgart)

Recollements are analogues of short exact sequences, deconstructing derived categories into 'smaller' ones. Iterating this procedure one may arrive at a stratification, which can be seen as an analogue of composition series. For some classes of algebras, we have obtained analogues of the Jordan–Hölder theorem for such situations. But in general, H.X. Chen and C.C. Xi have shown that the Jordan–Hölder theorem fails. Several classes of algebras have been shown to be 'derived simple'.

(This is based on joint work with Lidia Angeleri, Qunhua Liu and Dong Yang).

Support varieties for transporter category algebras

Fei Xu (Universitat Autónoma de Barcelona)

Let G be a finite group. Over any finite G-poset P we may define a transporter category as the corresponding Grothendieck construction. The classifying space of the transporter category is the Borel construction on the G-space BP, while the k-category algebra of the transporter category is the (Gorenstein) skew group algebra on the G-incidence algebra kP.

We introduce a support variety theory for the category algebras of transporter categories. It extends Carlson's support variety theory on group cohomology rings to equivariant cohomology rings. In the mean time it provides a class of (usually non selfinjective) algebras to which Snashall-Solberg's (Hochschild) support variety theory applies. Various properties will be developed. Particularly we establish a Quillen stratification for modules.

Silting objects, simple-minded objects and (co-)t-structures

Dong Yang (University of Stuttgart)

Let A be a finite-dimensional algebra over a field. Then there is bijection between

(i) silting objects,
(ii) simple-minded objects,
(iii) bounded t-structures of $D^b(\mathrm{mod}\, A)$ with length heart,
(iv) bounded co-t-structures of $K^b(\mathrm{proj} A)$.

I will present a proof and give a concrete example.

The Batalin–Vilkovisky structure over the Hochschild cohomology of a group algebra

Guodong Zhou (École Polytechnique Fédérale de Lausanne)

Let k be a field and let G be a finite group. It is well known that there is an isomorphism of graded vector spaces

$$HH^*(kG) \simeq \bigoplus_{x \in X} H^*(C_G(x))$$

where X is a set of representatives of the conjugacy classes of elements of G, and $C_G(x)$ is the centralizer of x in G. Following the work of Gerstenhaber and Tradler, there are a cup product, a Gerstenhaber Lie bracket and a Batalin–Vilkovisky structure over the Hochschild cohomology of kG. The goal of this work is to define these structures over the right side such that the above isomorphism is an isomorphism of BV-algebras. The case of cup product has been done by Siegel and Witherspoon, which in turn confirmed a conjecture of Cibils. We will illustrate our method by some examples. This talk is based on a joint work in progress with Yuming Liu.