# Abstracts

## Iteration of quasiregular maps

Bergweiler, Walter

Christian-Albrechts-Universität zu Kiel

**Abstract:** Sun Daochun and Yang Lo have extended the Fatou-Julia iteration theory of rational functions to quasiregular self-maps on the Riemann sphere. We shall report on some recent results concerning the dynamics of quasiregular maps in higher dimensions.

In particular, we will discuss joint work with Fletcher, Langley and Meyer concerning the escaping set $I(f) = \{x \in \mathbb{R}^d : f^n(x) \to \infty\}$ of quasiregular maps $f : \mathbb{R}^d \to \mathbb{R}^d$ with an essential singularity at $\infty$. We show that $I(f)$ is non-empty and in fact that $I(f)$ has an unbounded component. The corresponding results for entire functions are due to Eremenko as well as Rippon and Stallard.

We also show that certain results on the dynamics of exponential functions $E_\lambda(z) = \lambda e^z$ have three-dimensional analogues if the exponential map is replaced by a quasiregular map $F : \mathbb{R}^3 \to \mathbb{R}^3 \setminus \{0\}$ introduced by Zorich in 1967. This map is obtained by mapping the infinite "square beam" $\{(x, y, z) \in \mathbb{R}^3 : |x| < 1, |y| < 1\}$ quasiconformally onto the half space $\{(x, y, z) \in \mathbb{R}^3 : z > 0\}$, followed by reflections along sides of square beams and the plane given by $z = 0$.

Specifically, we will be concerned with the extension of a result of Devaney and Krych who showed in 1984 that if $0 < \lambda < 1/e$, then $E_\lambda(z)$ has an attracting fixed point for which the complement of the basin of attraction is a "Cantor set of curves". These curves are called *hairs*. Such a hair is a simple curve that connects a finite point – called the *endpoint* of the hair – to $\infty$. McMullen showed in 1987 that the union of the hairs has Hausdorff dimension 2. Karpińska then proved in 1999 that already the set of endpoints of the hairs has Hausdorff dimension 2, while the set of hairs without endpoints has Hausdorff dimension 1.

We show that for suitable parameters $a \in \mathbb{R}$ Zorich's map $f_a(x, y, z) = F(x, y, z) - (0, 0, a)$ has an attracting fixed point for which the complement of the basin of attraction is an uncountable union of hairs. The set of endpoints of these hairs has Hausdorff dimension 3, while the set of hairs without endpoints has Hausdorff dimension 1.

## The Schwarz-Pick lemma for plannar harmonic mappings

Chen, Huaihui

Nanjing Normal University

**Abstract:** The Schwarz-Pick lemma for harmonic mappings of the unit disk into itself is established completely.

## Non-existence of KAM torus

Cheng, Chongqing

Nanjing University

**Abstract:** Given an integrable Hamiltonian $h_0$ with $n$-degrees of freedom and a Diophantine frequency $\omega$, then, arbitrarily close to $h_0$ in the $C^r$ topology with $r < 2n$, there exists an analytical Hamiltonian $h\epsilon$ with no KAM torus of rotation vector $\omega$.

## A q-deformation of classical Nevanlinna theory

Chiang, Yik-Man

The Hong Kong University of Science and Technology

**Abstract:** We introduce a divided difference operator to rewrite the classical Nevanlinna theory in the complex plane. We will discuss this operator which is closely related to special functions, and some of the consequences derived from the theory. This is a joint work with Shaoji Feng.

## Theorem in different dimensions

Drasin, David

Purdue University

**Abstract:** *

## Gauss transformation and three multifractal spectra in continued fractions

Fan, Aihua

Purdue University

**Abstract:** Multifractal spectrum is usually described by the Legendre transform of a free energy function (sometimes called pression function) and is then a concave function. It is the case for the multifractal spectra of the Birkhoff averages on a nice dynamical system. We will show that new phenomena appear in the Gauss dynamics which is tightly related to continued fractions. This is illustrated by three multifractal spectra of Liapunov exponent, Khintchine exponent and frequency of the partial quotients.

## Hilbert-Polya space and Riemann zeta function

Ge, Liming

AMSS, CAS

**Abstract:** A Hilbert-Polya space is constructed on the positive real line. The (point) spectrum of the differential operator restricted to this space agrees with nontrivial zeros of Riemann zeta function. Some related questions are discussed.

## Value distribution theory and the research of Yang Lo

Hayman, Walter

Imperial College London

**Abstract:** Value distribution Theory is concerned with the position and frequency of solutions of the equation f(z)=w. Here f may be entire, i.e. an everywhere convergent power series or

meromorphic, i.e. the ratio of two such series or a function in some other domain, such as an angle or a disk. Yang Lo's significant contribution to this area and its relation to asymptotic values of f(z) will be highlighted. Some important contributions to normal families of functions in a finite disk will also be described.

## Gap phenomenon for proper holomorphic maps between classical domains

Huang, Xiaojun

Rutgers University

**Abstract:** In this talk, we survey some recent studies on various rigidity and gap property for proper holomorphic maps. We also formulate some open problems.

## Constructions of certain square-integrable automorphic functions

Jiang, Dihua

University of Minnesota

**Abstract:** It is one of the basic problems in the theory of automorphic representations to understand square-integrable automorphic functions. The Arthur trace formula and its refinements will produce parametrization of square-integrable automorphic functions (or representations) of classical groups in terms of these on the general linear groups. In this talk, we will discuss how to construct square-integrable (more importantly, cuspidal) automorphic functions (or representations) of classical groups in terms of these on the general linear groups. If time is permitted, we will also discuss some related problems in the theory of automorphic forms in the framework of the Langlands Program.

## Lifting Problem and Holomorphic Motions

## in Complex Analysis

Jiang, Yunping

Queens College of CUNY

**Abstract:** I will talk a joint work with Sudeb Mitra and Zhe Wang about the lifting problem in Teichmuller theory. We study liftings of holomorphic maps into some Teichmuller spaces and give a complete proof of the lifting problem for Teichmuller spaces of closed sets in the Riemann sphere. We also study the relationship between universal holomorphic motions and holomorphic lifts into Teichmuller spaces of closed sets in the Riemann sphere.

## Cantor Boundary Behavior of Analytic Functions

Lau, Ka-Sing

The Chinese University of Hong Kong

**Abstract:** There is a vast literature to study the boundary behavior of univalent functions on the unit disc $\mathbb{D}$ in connection with the Riemann mapping theorem. Our interest is on the class of analytic functions *f(z)* for which the image curves $f(\partial\mathbb{D})$ form infinitely many (fractal) loops; they are not univalent of course. We formulated this as the Cantor boundary behavior (CBB). We develop a general theory of this property in connection with the analytic topology, the distribution of the zeros of $f'(z)$ and the mean growth rate of $f'(z)$ near the boundary. Among the many examples, we showed that the complex Weierstrass functions have the CBB. Also we study the CBB for the Cauchy transform of the canonical Hausdor measure on the Sierspinski gasket, which is the original motivation of this investigation raised by Strichartz.

## On analytic properties of L-functions

Li, Baoqin

International University Miami

**Abstract:** L-functions, with the Riemann zeta-function as the prototype, are Dirichlet series satisfying certain axioms, which are important objects in number theory. We will discuss some

recent results on analytic properties of L-functions, especially on distribution of c-values of L-functions.

## Positivity of central values of L-functions

Li, Jian-Shu

The Hong Kong University of Science and Technology

**Abstract:** Let $L(s, \pi)$ be the "standard" L-function of an automorphic cuspidal representation of the general linear group. Then, as predicted by the generalized Riemann hypothesis, the central value $L(\frac{1}{2}, \pi)$ should be non-negative. In this talk we shall outline a new proof of this fact in the case when $\pi$ arises from a cohomological cuspidal automorphic form of a suitable unitary group. Our ultimate goal is to obtain an exact formula for the central value.

## Rigidity for solutions of the degenerate

### elliptic differential operators

Li, Songying

University of California, Irvine

**Abstract:** In talk, I will represent some rigidity results on the solutions of some degenerate elliptic differential operators, which include the Laplace-Beltrami operators in metric of Bergman type, complex Monge-Ampère operators, and proper harmonic mappings.

## The explicit formula and Connes' trace formula

Li, Xianjin

Brigham Young University

**Abstract:** I will discuss A. Connes' approach to RH. In particular, some of my results related to the explicit formula and Connes' trace formula will be presented.

## Topological Vorticity and Conserved Geometric Motion

Lin, Fang-Hua

New York University

**Abstract:** *

## Recen results on moduli spaces

Liu, Kefeng

University of California, Los Angeles

**Abstract:** I will discuss recent results on the geometry and topology of moduli spaces and Teichmuller spaces of Riemann surfaces and Calab-Yau manifolds.

## Analytic continuation of holomorphic maps by means of the Bergman kernel

Mok, Ngaiming

The University of Hong Kong

**Abstract:** For the study of holomorphic maps $f : D \to \Omega$ between bounded symmetric domains in their Harish-Chandra realizations, first results were obtained in the 70s concerning rigidity of locally defined maps (near some boundary point) due to Alexander in the case of the unit ball and due to Henkin-Tumanov in the higher rank case. In this talk we will focus on recent work on germs of holomorphic maps $f : (D; 0) \to (\Omega; 0)$ with special geometric properties such as holomorphic isometries and measure-preserving holomorphic maps. Our discussion will involve (a) extension results on germs of holomorphic isometries between bounded domains with algebraic Bergman kernels, such as bounded symmetric domains; (b) characterization of measure-preserving algebraic correspondences as modular correspondences: solution of a problem raised by Clozel-Ullmo (joint work with Sui-Chung Ng). Our methods incorporate on

the one hand extension techniques in Several Complex Variables such as algebraicity results due to S. Webster and Xiaojun Huang on CR-maps and the results of Alexander and Henkin-Tumanov. We also develop techniques resembling those for Segre varieties to force analytic continuation using holomorphic polarization of functional identities arising from the Bergman kernel and study deformation of solutions of these identities. Our results illustrate the general philosophy that holomorphic maps between bounded domains are expected to be as regular as their Bergman kernels whenever they preserve invariant $(p, p)$-forms arising from the Bergman metric.

## On Critical Exponent of Free Energy on Julia Set

Qiao, Jianyong

China University of Mining and Technology

**Abstract:** We deal with the analytical continuation in the complex plane of free energy of the Potts model on diamond-like hierarchical lattices. The singularities of free energy of this model lie on the Julia sets of a family of rational mappings. We study the asymptotics of the complex critical exponent of the free energy when the temperature goes along hyperbolic geodesics to the boundary of the Fatou set.

## A reverse Denjoy theorem

Rossi, John

Virginia Polytechnic Institute and State University

**Abstract:** Suppose that $C_1$ and $C_2$ are two simple curves joining 0 to $\infty$, non-intersecting in the finite plane except at 0 and enclosing a domain $D$ which has angular measure at most $2\alpha$ $(0 < \alpha < \pi)$ for all large r. Suppose also that u is a non-constant subharmonic function in the plane such that u(z) $=B(|z|,u):=\sup$ $\{u(z):|z|=r\}$ for all large z$\in$ $C_1 \cup C_2$ Let $A_D(r,u)=\inf\{u(z):z\in D\cap\{|z|=r\}\}$. It is shown that if $A_D(r,u)=O(1)$ then $\liminf_{r\to\infty} B(r,u)/r^{\pi/2\alpha} > 0$.

## Some recent develpement in Nevanlinna theory and its application

Ru, Min

Huston University

**Abstract:** In this talk, I'll report some works I have done in recent years in Nevanlinna theory and its applications. In partcular I'll outline the proof of my recent result of the Second Main Theorem for Holomorphic mappings into algebric varieties intersecting hypersurfaces, as well as pplications in the study of Gauss map of minimal surfaces, and in Diophantine approximations in number theory.

## On one-dimensional dynamics with weak hyperbolicity

Shen, Weixiao

University of Science and Technology of China

**Abstract:**We shall discuss equality of fractal dimensions, physical measures and mixing speed of one-dimensional real and complex dynamical systems under weak hyperbolcity assumptions. This talk is based on a joint work with Juan Rivera-Letelier.

## Dynamics around irrationally indifferent fixed points of holomorphic functions

Shishikura, Mitsuhiro

Kyoto University

**Abstract:** A fixed point of a holomorphic mapping of one variable is called irrationally indifferent if the derivative (multiplier) has modulus one without being a root of unity. Such a fixed point leads to interesting dynamical phenomena and a complicated invariant set especially when the fixed point is not linearizable. We focus on the case where the rotation number is of high

type (the continued fraction expansion has large coefficients), and analyze the dynamics via near-parabolic renormalization. For a certain class of maps, including quadratic polynomials, we give a description of invariant sets around the fixed point.

## Techniques of deformation in hyperbolicity problems

Siu, Yum-Tong

Harvard University

**Abstract:** We will discuss the application of the techniques of deformation to hyperbolicity problems in value distribution theory of several variables such as the hyperbolicity of generic hypersurfaces of high degree in the complex projective space and the moduli space of canonically polarized manifolds.

## Quasisymmetric maps a geometrical viem

Wu, Jang-Mei

University of Illinois at Urbana-Champaign

**Abstract:** Quasisymmetric maps are generalizations of conformal maps and quasiconformal maps to general metric spaces. The theory of quasisymmetric maps has recently found applications in complex dynamics, geometric group theory and structure of manifolds. However a large number of fundamental questions remain unanswered, for example Quasisymmetric parametrization of metric spaces by Euclidean spaces, Factorization of quasisymmetric maps into maps of small distortion, Extension of quasisymmetric maps to an ambient space. We will survey some recent progress, and illustrate the mysteries by examples from geometric topology.

## On angular distributions of meromorphic functions

Wu, Shengjian

Peking University

**Abstract:** The talk consists of three parts: (1) An introduction to the theory of angular distribution of meromorphic functions. (2) Some results on the relationship between the growth of meromorphic functions and angular distributions of meromorphic functions. (3) Some applications of the theory of angular distributions of meromorphic functions to the complex oscillation theory.

## Twisted torsion of de Rham and Dolbeault complexes

Wu, Siye

Colorado-Boulder & HKU

**Abstract:** Associated to a topological space there are many invariants. One of them is torsion. It can be defined combinatorially by simplicial method or analytically by zeta-function regularisation. In this talk, I will explain how to generalise the analytic torsion by adding a flux form motivated from physics as well as its holomorphic analogue. This is a joint work with Mathai.

## On A Transonic Shock Problem and Mixed Type Equations

Xin, Zhouping

The Chinese University of Hong Kong

**Abstract:** In this talk, I will discuss some recent progree on the studies of the transonic shock problem in a finite varaible nozzle proposed by Courant-Friedrichs. This problem can be reduced to study a nonlinear free boundary value problem for the steady compressible Euler equations which a mixed type system (hyperbolic-elliptic). Our analysis depends crucially on solving a nonlocal elliptic system with a free parameter. Some other related problems will be discussed also.

## Metrics on complex manifolds

Yau, Shing-Tung

Harvard University

**Abstract:** *

## Explicit construction of moduli space of bounded complete Reinhardt domains in $C^n$ and Hilbert 14th problems

Yau, Stephen Shing-Toung

University of Illinois at Chicago

**Abstract:** One of the most fundamental problems in complex geometry is to determine when two bounded domains in $C^n$ are biholomorphically equivalent. Even for complete Reinhardt domains, this fundamental problem remains unsolved for many years. Using the Bergmann function theory, we construct an infinite family of numerical invariants from the Bergman functions for complete Reinhardt domains in $C^n$. These infinite family of numerical invariants are actually a complete set of invariants if the domains are pseudoconvex with $C^1$ boundaries. For bounded complete Reinhardt domains with real analytic boundaries, the complete set of numerical invariants can be reduced dramatically although the set is still infinite. We shall also discuss the role of the Hilbert 14th problem in the construction of numerical biholomorphic invariants of complete Reinhardt domains in $C^n$.

## On Nevanlinna theory of meromorphic functions on annuli

Ye, Zhuan

Northern Illinois University

**Abstract:** We discuss the recent development of Nevanlinna theory of meromorphic functions on annuli, which extends results in Nevanlinna theory in complex plane or in disk. In particular, we define Nevanlinna counting, proximity and characteristic functions with two independent

variables for functions meromorphic on annuli and show analogues of many well-known theorems, including Jensen's formula, logarithmic derivative lemmas and the first and the second main theorems. Our approach taken on annuli is a unified treatment of functions meromorphic in the complex plane, disk and annulus. It allows one to obtain many results in the complex plane and in disk as corollaries of our results in annuli.

## Proof of the Branner-Hubbard conjecture and some applications

Yin, Yongcheng

Fudan University

**Abstract:** By means of a nested sequence of some critical pieces constructed by Kozlovski, Shen, and van Strien, and by using a covering lemma recently proved by Kahn and Lyubich, we prove that a component of the filled-in Julia set of any polynomial is a point if and only if its forward orbit contains no periodic critical components. It follows immediately that the Julia set of a polynomial is a Cantor set if and only if each critical component of the filled-in Julia set is aperiodic. This result was a conjecture raised by Branner and Hubbard in 1992. Some applications will also be given.

## Variations on Montel's Theorem

Zalcman, Lawrence

Bar Ilan University

**Abstract:** A survey of various generalizations and strengthenings of the classical theorem of Montel, with an emphasis on surprising recent improvements.

## Polarized endomorphisms and preperiodic points

Zhang, Shou-Wu

Columbia University

**Abstract:** In this talk, I will propose several problem about endomorphisms with same set of preperiodic points including a repaired dyanamical Manin–Mumford conjecture.